INFLATIONARY LAMBDA-UNIVERSE WITH TIME-VARYING FUNDAMENTAL CONSTANTS

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Abstract

We solve the Universe for dark energy represented by a variable cosmological constant, along with a JBD (Jordan-Brans-Dicke) model with tine varying speed of light, entailing variable fine structure constant. Inflation is taken as the prevailing scenario, but we provide food for thought, to discuss how this model with fundamental constants applies to the present accelerating phase. Along with a curious discussion of a possible Planck´s time not coincident with ten to the power -43, in seconds, this model is full of novelties.

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Dark energy is possibly represented by a time-varying cosmological "constant". On the other hand, the fact that the equation of state of the present Universe is very near $p = -\rho$, where p, ρ are cosmic pressure and energy density, makes us think that inflationary scenario with exponential scale factor could not only be of importance as early Universe phase, but could also be important for the present accelerating Universe, because the deceleration parameter would be negative, as required by modern developments in the observational front. The present paper will deal with such questions. We extend here the results of J.D.Barrow , who in a series of papers authored by him alone, or with collaborators, dealt with variable "constants" , cosmological models, including fine structure "constant" , and inflationary scenario cosmologies [1]; we calculate the possibility of different than the accepted values for Planck´s Universe quantities (like Planck's time or Planck's energy, etc), due to the effects of inflation on such time variations.

Weinberg [14], has reviewed modern cosmology, and we refer to him for updated information. Riess et al [3] found evidence for an accelerating Universe with an observed deceleration parameter average near -1 , by means of Supernovae observations. Not only were placed constraints on the Hubble´s constant but also on the mass density, the cosmological constant (i.e, the vacuum energy density), the dynamical age of the Universe, and most important in our opinion, the deceleration parameter. In the abstract of one of the papers published by Riess and his group, there is a comment that eternally expanding models with positive cosmological constants are favoured unanimously. In the present paper, we shall find a theoretical model of the Universe that could eventually point to such an eternal Universe with infinite age ($-\infty < t < \infty$), or else it could be understood as defining an exponential inflationary phase for the early Universe, when according to our calculations the fine structure constant should have a huge value, when compared with its present value ($\alpha_0 \approx 1/137$). Both possibilities render useful the study of $q = -1$ cosmologies.

On the other hand, Webb et al [4] found evidence, in the spectra of distant quasars, for a time varying fine structure constant, spanning 23% to 87% of the age of the Universe, if we suppose that there was a big bang, say about 18 billion years ago. Webb et al. found that α was smaller in

the past than it is today. This could be explained by time variations of at least one of the constants that appear in its definition(the electron charge, the speed of light or Planck´s constant). They also comment that a common property of unified theories, when applied to cosmology, is that they predict time-space dependence of the coupling constants.

We shall show how to accommodate both results into a JBD (Jordan-Brans-Dicke) framework, where the speed of light is also variable, as in Barrow [5]. John D. Barrow [1] analyzed generalizations of General Relativity that incorporates a cosmic time variation of the velocity of light in vacuum and the Newtonian gravitational constant G, as proposed by Albrecht and Magueijo [6]. He found exact solutions for Friedmann Universes and determined the rate of variation of c required to solve the flatness and classical cosmological constant problems. Potential problems with this approach were discussed by Barrow, and Jordan-Brans-Dicke solutions were also presented.

Barrow [7] has pointed out the possible relevance of scalar-tensor gravity theories in the study of the inflationary phase during the early Universe. He obtained exact solutions for homogeneous and isotropic cosmologies in vacuum and radiation cases, for a variable coupling "constant" , $\omega = \omega(\phi)$, where ϕ stands for the scalar field. Although classical standard Big Bang scenario imposes entropy conservation, in 1981 A. Guth [8] proposed that the flatness along with the monopole and the horizon problems could be made to disappear if the Universe traversed an epoch with negative pressure and terminated it in a huge entropy increase (old inflation).Many new models were afterward been invented, in particular the new inflationary, Linde´s chaotic version of it [9]and extended inflation[1]. Berman and Som [10] further have shown that in Brans-Dicke theory [11]we can accommodate an inflationary exponential phase with positive pressure. For accounts on inflation, see, for instance, Linde's book[9] or the updated Weinberg [14].

We define the deceleration parameter, in terms of the scale factor and its time derivatives as:

$$q = -\frac{\ddot{R}R}{\dot{R}^2}. \tag{1}$$

It is evident that an exponential scale factor, like

$$R = R_0 e^{Ht} \tag{2}$$

(R_0 and H constants) yields the desired result, i.e.

$$q = -1. \tag{3}$$

Before introducing the cosmological "constant", we first work with Barrow´s equations , which do not contain a Λ term and zero tricurvature:

$$H^2 = \frac{8\pi\rho}{\phi} - \frac{\dot{\phi}}{\phi}H + \frac{\omega}{6}\frac{\dot{\phi}^2}{\phi^2} \tag{4}$$

$$\dot{\rho} + 3H\left(\rho + \frac{p}{c^2}\right) = 0 \tag{5}$$

$$\ddot{\phi} + 3H\dot{\phi} = \frac{8\pi}{(3+2\omega)}\left(\rho - \frac{3p}{c^2(t)}\right) \tag{6}$$

where ρ, p, ϕ, ω and k stand respectively for energy density, cosmic pressure, scalar field, coupling constant, and tricurvature. This applies to a Robertson-Walker´s metric. Equation (5) represents the energy-momentum tensor conservation, for the original matter fluid.

We find the following solution:

$$\phi = \phi_0 e^{\beta t} \tag{7}$$

$$\rho = A\phi_0 e^{\beta t} \tag{8}$$

$$\beta = -3H(1+\gamma) \tag{9}$$

$$c = c_0 e^{\delta H t} \tag{10}$$

subject to the following conditions :

$$\frac{p}{c^2} = \gamma\rho \tag{11}$$

$$H^2 = -\frac{8\pi A}{\left[2 + 3\gamma + \frac{3}{2}\omega(1+\gamma)^2\right]} \tag{12}$$

$$A = \frac{(3+2\omega)(\beta+3H)\beta}{(1-3\gamma)}. \tag{13}$$

We remark that the last three conditions were obtained by plugging our solution into the three field equations obtained by Barrow [5]. It is better to restrict our study to the case $k = 0$, in order that we need not bother with possible $\dot{c}$ terms in the above field equations.

If we introduce a cosmological term $\Lambda = \Lambda(t)$, we resort to Barrow-Bertolami theorem given by Berman[12] [13] which assumes the form of replacement:

$$\rho \rightarrow \rho + \Lambda/(8\pi)$$

,

$$p/c^2 \rightarrow p/c^2 - \Lambda/(8\pi) + \dot{\Lambda}\phi/(12\pi\dot{\phi})$$

.In such case, we find a reasonable solution, namely:

$$\Lambda = \Lambda_0 e^{\beta t}$$

where Λ_0 = constant. If we plug the solution back, we find that, instead of conditions (12) and (13) we must make the following replacement:

$$A \rightarrow A + \Lambda_0/(8\pi)$$

and we must have,

$$c_0^2 = -3\phi_0\gamma$$

. Likewise, as $c_0^2 > 0$, and , for normal situations, $\phi_0 > 0$ (so that, gravitation is attractive, i.e. , $G > 0$) we must have $\gamma < 0$. The weak energy condition is represented by a positive energy density, so that we must have a negative cosmic pressure. This is tantamount to what is admitted by the most recent observations of the present Universe.

On observational grounds, we impose that, for the present Universe

$$p = \epsilon\rho \tag{14}$$

, where $\epsilon \leq -0.9$. The speed of light must be adjusted in order that for the present Universe, we get the result 300,000 km/s . From relation (8) , the apparent desire for a decreasing energy density, makes us impose that $\beta < 0$. From (9) , we would need in such case, to impose that ,

$$1 + \gamma > 0 \tag{15}$$

Because we define the fine structure "constant" as

$$\alpha \equiv \frac{e^2}{\frac{h}{2\pi}c} \tag{16}$$

we find that

$$\frac{\dot{\alpha}}{\alpha} = -H\delta \tag{17}$$

where of course, H stand for Hubble ‘s constant for this model. This means that α was exponentially larger in the early Universe than it is today, if $\delta > 0$. Good discussions on fine strucuture time-varying "constant", either through $\dot{c} \neq 0$ or through $\dot{\epsilon}_0 \neq 0$ can be found in Berman [15] [16].

Planck´s quantities for energy length , mass and time could be drastically changed for the very early Universe (Grand Unification Epoch) as a consequence of the time variations of G and c. Take for example Planck´s length:

$$\lambda = \left(\frac{hG}{2\pi c^3} \right)^{1/2} \tag{18}$$

We find that, with $G \propto \phi^{-1}$,

$$\lambda \propto e^{-(1/2)[\beta+3H]t} \tag{19}$$

On the other hand, Planck´s time is λ/c so that it would vary like:

$$t_p \propto e^{(3/2)H[1+\gamma-\delta]t} \tag{20}$$

This means that with most probability $t_p = 10^{-43}$ seconds is no longer a valid result. For the particular case $\delta = 1 + \gamma$, we recover a constant Planck´s time. The condition $\delta > 0$ means that $\gamma > -1$, for constant t_P .

Even with exponential equations for ρ and p , we could adjust constants in order that an equation of state be obtained such that $p = \frac{1}{3}\rho$, in case we would like to include a radiation phase, during the evolution of the Universe.

As the reader can check, many questions can be posed over this mathematical and theoretical model. . May be we are just in face of an eternal Universe model, of the type studied in the golden days of stationary state models. If this is not the case, we have nevertheless found a model that deserves attention, and that can point to very interesting developments in the study of the very early Universe Physics. Finally, our model has zero-pressure, zero-density and zero-lambda when $t \to \infty$. This model deserves the qualification of present Universe validity, but also has to do with the very early Universe inflation, but also applies with minor modification, to a radiation phase. The novelties in this paper, deserve attention.

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